Pixel Noise Correction by ERA method for Precise weak lensing shear measurement

Yuki Okura
RIKEN

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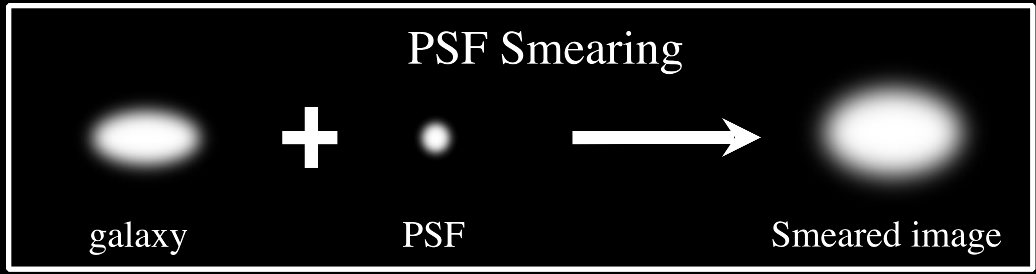
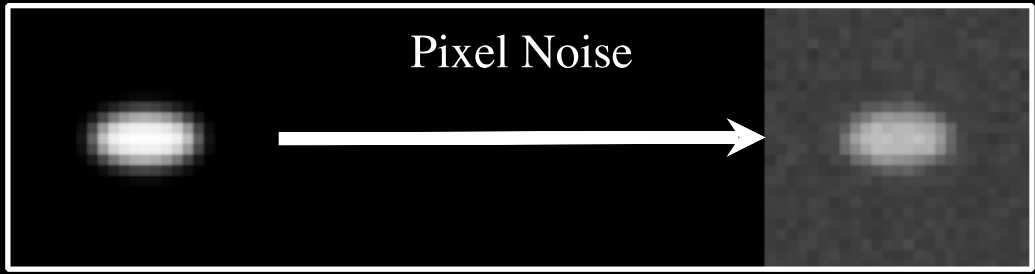
I’m developing a new weak leasing shear method named ERA method for precise weak lensing analysis like strong cosmological parameter constrain via cosmic shear measurement. This is a study about pixel noise which makes systematic error in shear measurement and its correction.

Introduction :

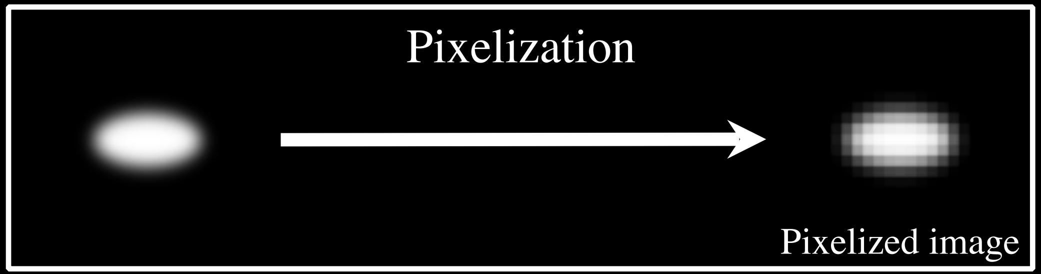
HSC survey requires lower than 1% systematic error for shear measurement, and so ellipticity of galaxies, but many effects changes ellipticity, so we need to measure galaxy shapes with some corrections



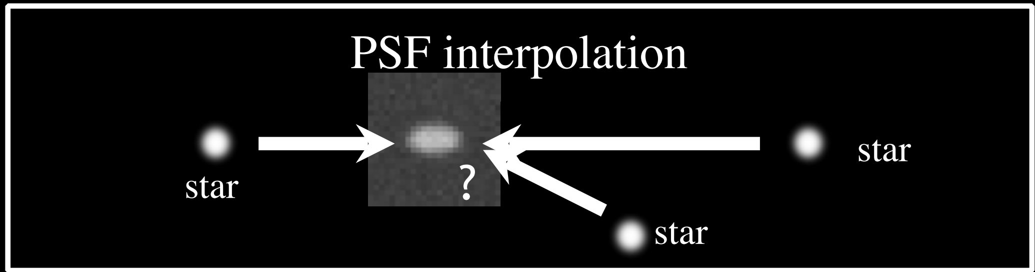
Galaxy image is smeared by atmosphere. This is corrected by using PSF information measured from surrounding stars.



Galaxy image has extra random count from Poisson noise of sky brightness. Systematic effect can be predicted statistically. —— This study.



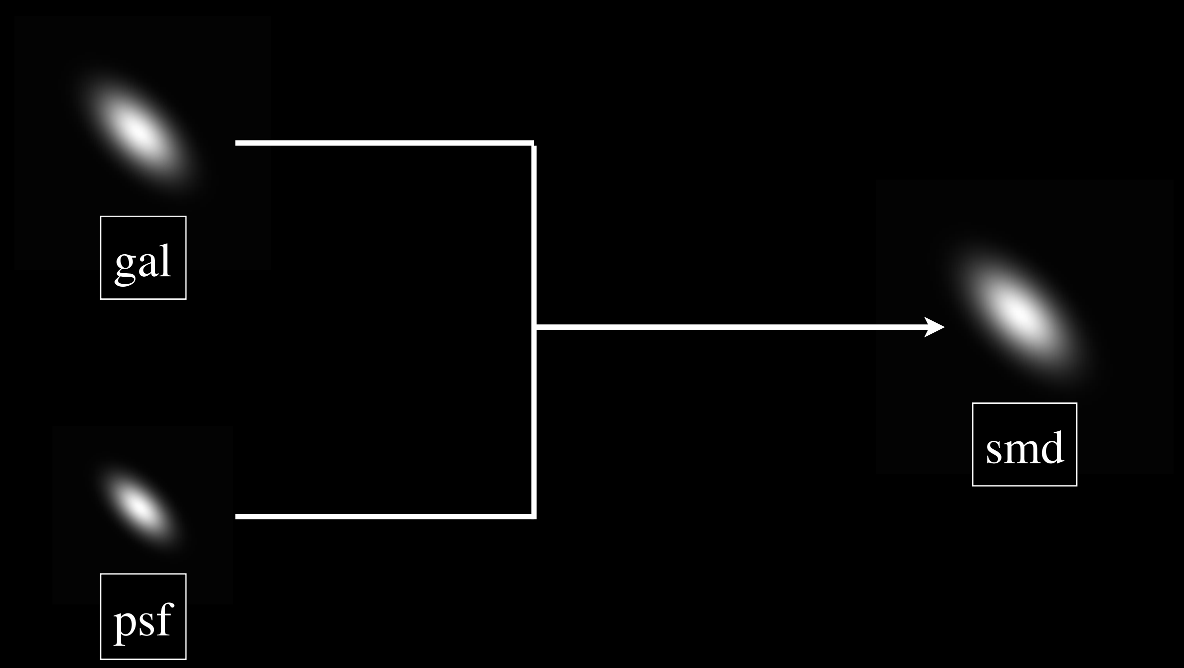
Galaxy image is observed by pixel units, so the shape changes pixel units shape.



PSF information on galaxy is estimated from surrounding stars, with some interpolation method, so it's not perfect.

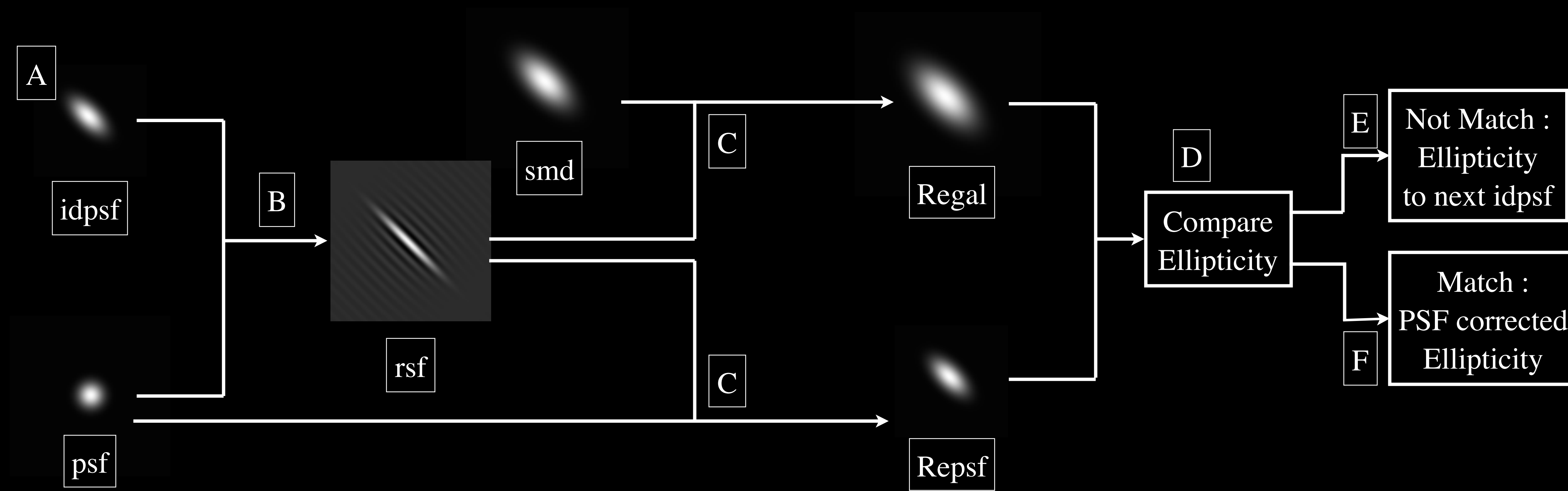
Basic idea for PSF correction :

The Basic idea is that if PSF has ellipticity same as galaxy, PSF smearing does not change ellipticity, so we don't need to correct PSF effect. But, in real analysis, the ellipticity of galaxy and PSF is not same. ERA method re-smears PSF image and galaxy image artificially by re-smearing function to make idealised PSF and to make re-smeared galaxy which has same ellipticity as before PSF smearing. Ellipticity of the re-smeared galaxy is PSF corrected ellipticity. In this correction, there are no approximations, so no systematic error. Ellipticity with any definition can be used.



If galaxy and pdf has same ellipticity, smeared galaxy has same ellipticity

Basic idea for PSF correction :



Steps for measuring PSF corrected ellipticity
PSF correction in ERA method is finding RSF which re-smears galaxy and pdf and the re-smeared images have same ellipticity.

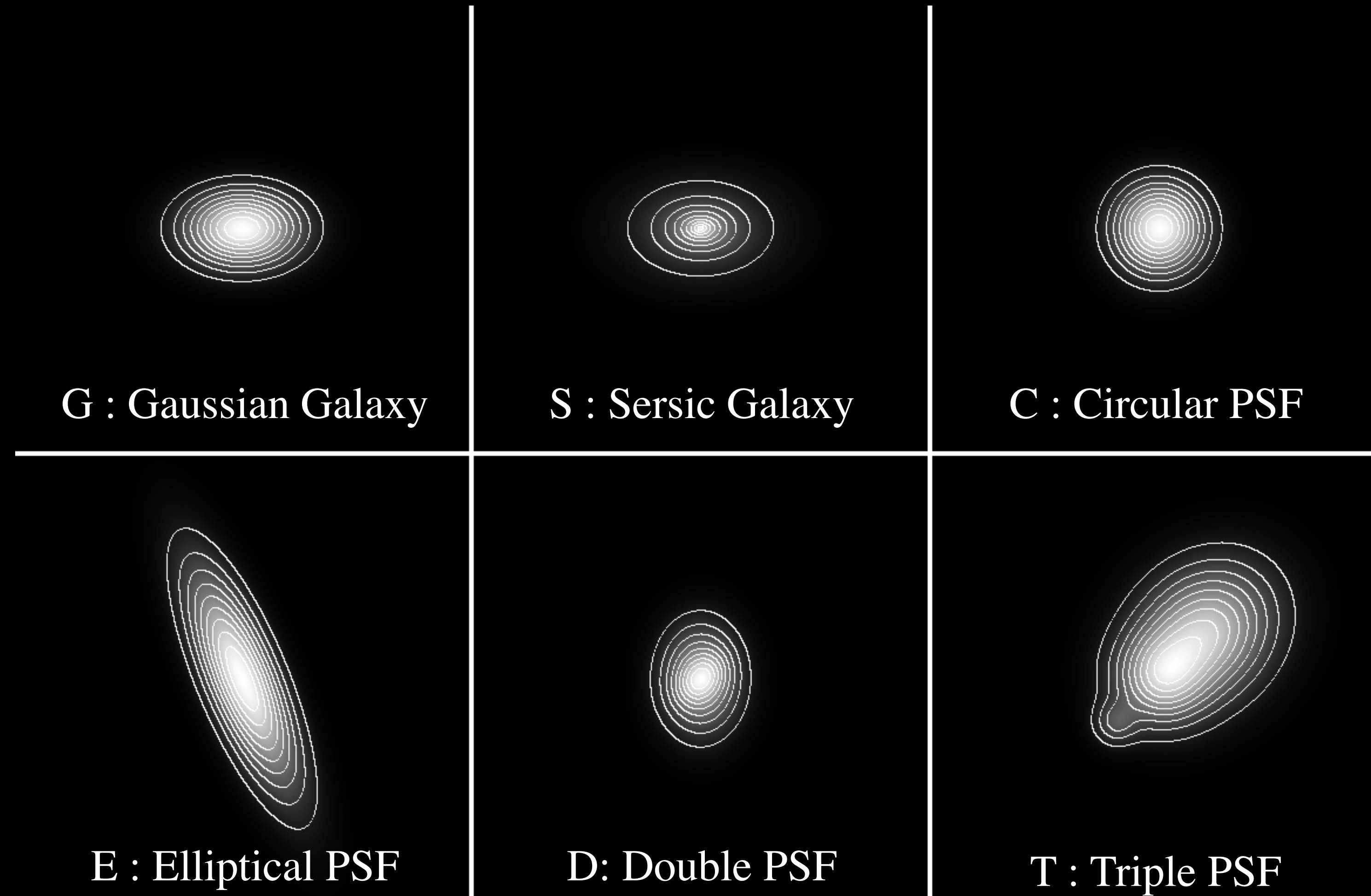
A : Making idealised PSF image with temporal ellipticity
B : Making Re-smeared Function(rsf) from idpsf and pdf by deconvolution.
C : Making Re-smeared galaxy and Re-smeared psf images
D : Measuring ellipticity of the two Rte-Smeared images
E : If the two ellipticities are not match, the ellipticity is used for next temporal ellipticity -> A.
F : If the two ellipticities are match, it is PSF corrected ellipticity.

Simulation Test of PSF correction :

This is test for PSF correction, so
  No Pixel Noise
  No Pixelization

Simulation with 2 types of galaxies (Gaussian and Sersic) and 4 types of psf(Combination of Gaussian).

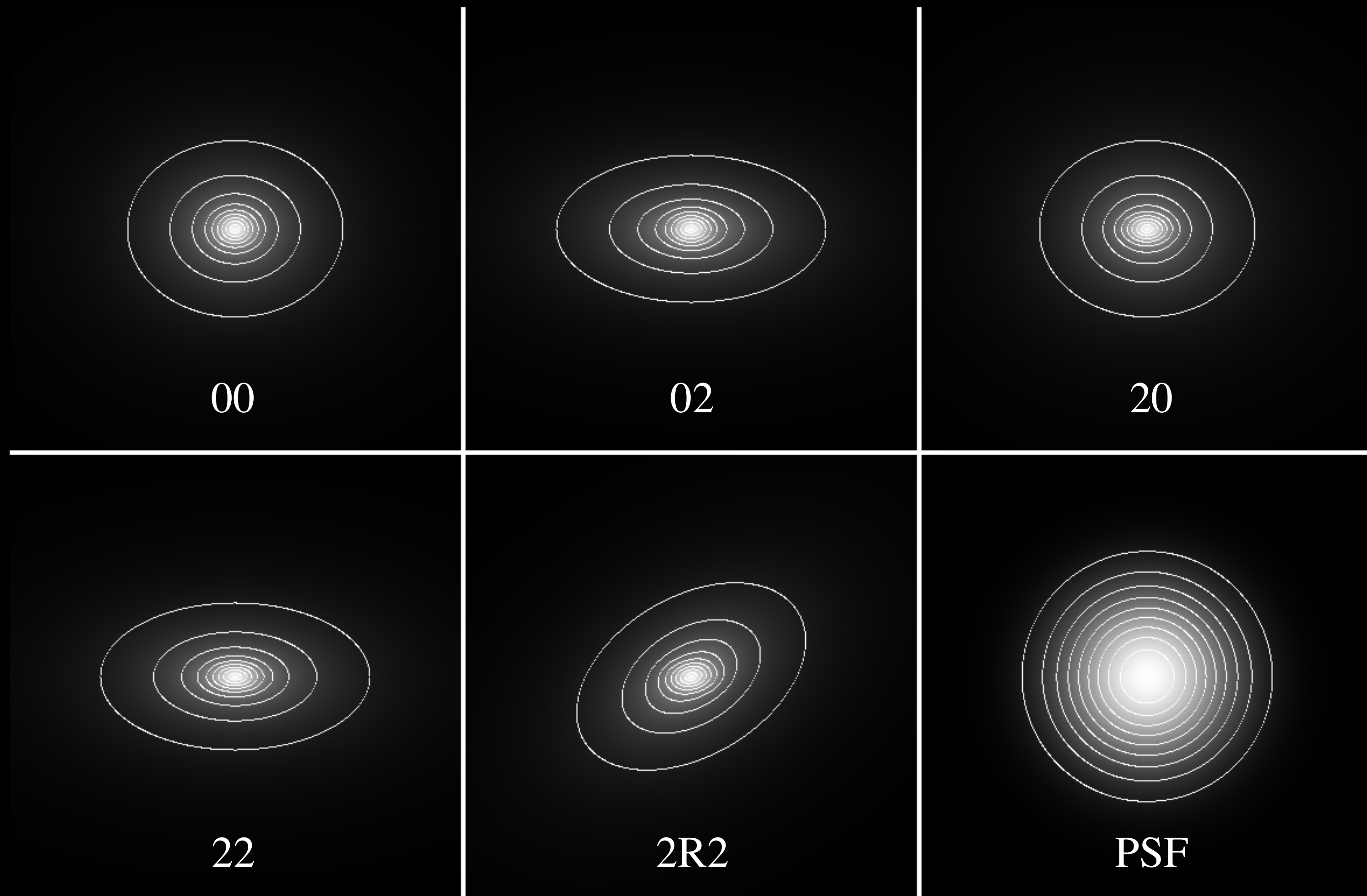
The systematic error ratio is very small ~ 10^-6.



Galaxy ID	Type PSF	r_{PSF}[pixels]	r_{IPSF}[pixels]	$\Delta\epsilon_{2nd}$	$\Delta\epsilon_{0th}$
G	C	100.0	150.0	1.343e-7	8.764e-8
G	E	100.0	350.0	-2.425e-6	-2.349e-6
G	D	90.2	150.0	-2.973e-7	-4.256e-7
G	T	150.0	275.0	-3.260e-6	-9.045e-6
S	C	100.0	150.0	-6.765e-8	5.201e-7
S	E	100.0	350.0	2.483e-8	-3.050e-7
S	D	90.2	150.0	5.747e-7	-3.376e-7
S	T	150.0	275.0	-8.324e-7	-1.757e-6

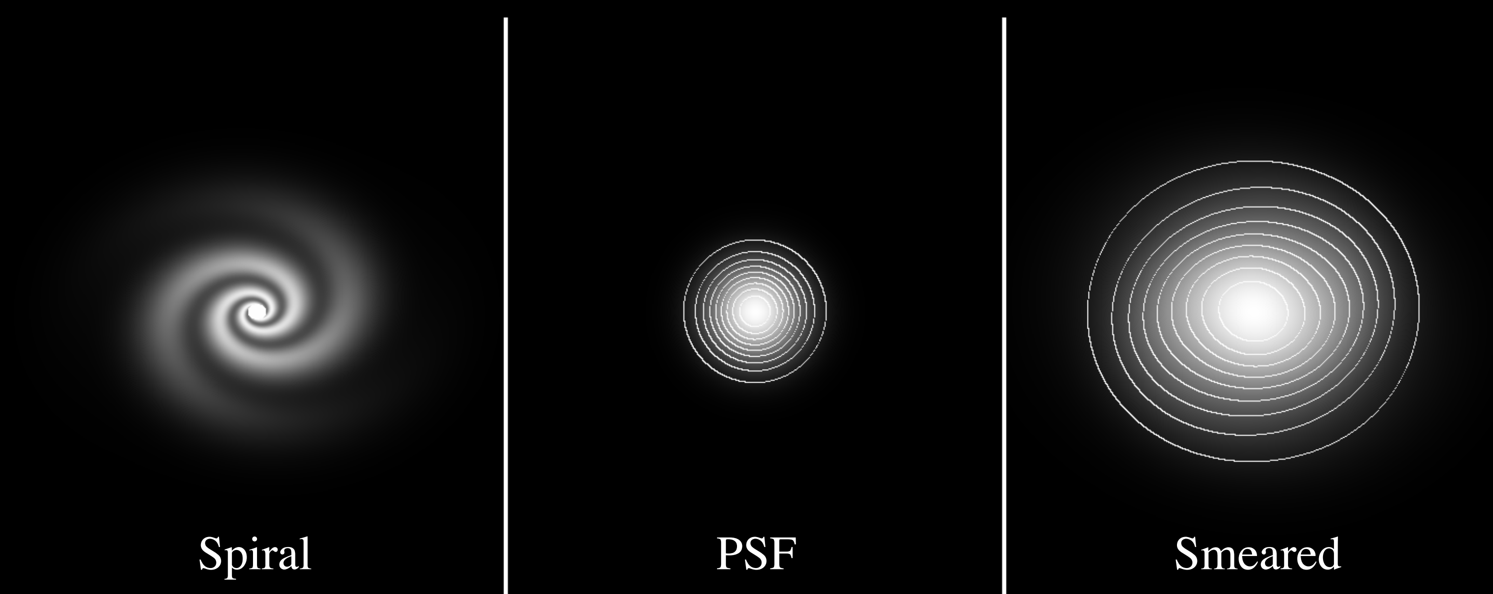
Table 3: The systematic error in shear measurement. The radius of PSF is best radius of weight function for measuring PSF shape with elliptical Gaussian weight.

Simulation Test of PSF correction :



Galaxy ID	$\Delta\gamma_{GAL}$	$\Delta\gamma_{2nd}$	$\Delta\gamma_{0th}$
00	3.254e-8	-7.225e-7	1.029e-5
02	1.455e-8	-5.661e-5	-1.067e-4
20	-1.230e-7	-7.182e-7	1.031e-5
22	-9.077e-9	-8.600e-7	-6.371e-6
2R2	3.087e-7	-4.277e-7	1.115e-5
spiral	-6.286e-6	-8.332e-5	-1.259e-4
multi	-1.536e-8	-3.387e-5	-5.264e-5

Table 5: The systematic error in the shear measurement from the double elliptical Galaxies.



Simulation with galaxies which have radial depended ellipticity.

The systematic error ratio is ~ 10^-5.

Basic idea for Pixel Noise correction :

Pixel noise is Poisson noise of sky count, so its count is random, but the ellipticity is measured with non-linear of image, so it makes systematic error.

Image moments and ellipticity is measured as

$$\mathcal{M}^N_M(I,\epsilon) = \int d^2\theta I(\boldsymbol{\theta})\boldsymbol{\theta}^N_M W(\boldsymbol{\theta},\epsilon) \qquad \epsilon(I) = \frac{\mathcal{M}^2_2(I,\epsilon)}{\mathcal{M}^2_0(I,\epsilon)}$$

then PSF corrected ellipticity is measured with Re-smeared galaxy image as

$$\epsilon(I^{Resmd}) = \frac{\mathcal{M}^2_2(I^{Resmd},\epsilon)}{\mathcal{M}^2_0(I^{Resmd},\epsilon)}$$

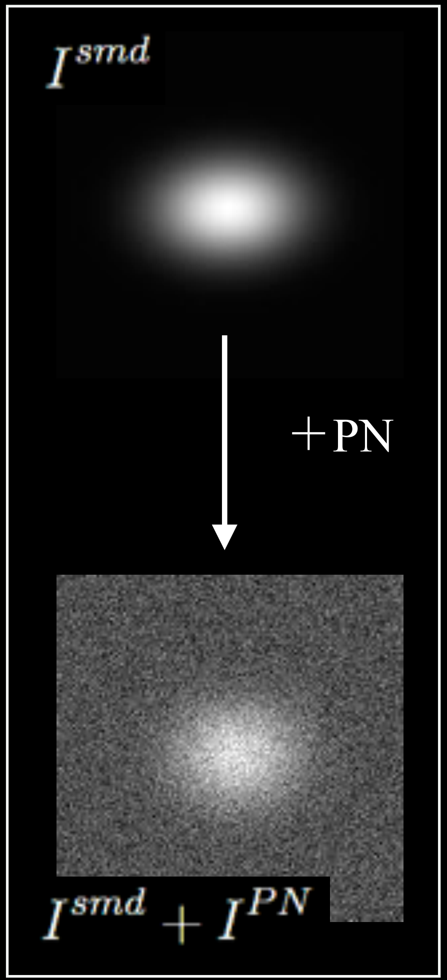
$$I^{Resmd} = I^{smd} * I^{rsf}(\epsilon) = \left(I^{gal} * I^{psf}\right) * \left(I^{idpsf}(\epsilon) \otimes I^{psf}\right)$$

Pixel Noise makes additional count and so additional ellipticity

$$I^{smd} \to I^{smd} + I^{PN} \qquad \epsilon \to \epsilon + \Delta\epsilon$$

PSF corrected ellipticity with pixel noise is measured as

$$\epsilon(I^{Resmd}) + \Delta\epsilon = \frac{\mathcal{M}^2_2(I^{Resmd} + I^{RePN},\epsilon+\Delta\epsilon)}{\mathcal{M}^2_0(I^{Resmd} + I^{RePN},\epsilon+\Delta\epsilon)}$$



Basic idea for Pixel Noise correction :

The idea of pixel noise correction is calculating 1st and 2nd order of the additional ellipticity by Taylor expansion of the equation of PSF corrected ellipticity

$$W(\epsilon + \Delta\epsilon) \approx W(\epsilon) + \Delta\epsilon \frac{\partial W}{\partial \Delta\epsilon} + ...$$

$$I^{rsf}(\epsilon + \Delta\epsilon) \approx I^{rsf}(\epsilon) + \Delta\epsilon \frac{\partial I^{rsf}}{\partial \Delta\epsilon} + ...$$

$$\Delta\epsilon = \Delta\epsilon_{(1)} + \Delta\epsilon_{(2)} + ...$$

1st order is linear to $\mathcal{M}^N_M(I^{RePN}, \epsilon)$, so the average is 0

$$\left\langle \Delta\epsilon_{(1)} \right\rangle = 0$$

Variance of 1st order is not 0, $\langle \mathcal{M}^N_M(I^{RePN}, \epsilon)\mathcal{M}^O_P(I^{RePN}, \epsilon) \rangle \propto \frac{1}{\mathrm{SNR}^2}$ and it is used for calculating weight

$$\left\langle |\Delta\epsilon_{(1)}|^2 \right\rangle = \sigma^2_{PN} \qquad w = \frac{\sigma^2_{int}}{\sigma^2_{int} + \sigma^2_{PN}}$$

Average of 2nd order is systematic error

$$\left\langle \Delta\epsilon_{(2)} \right\rangle = \text{bias}$$

Simulation Test of Pixel Noise correction : profile, radius and ellipticity

Taylor expansion approach must be good performance in low noise limit.

Galaxy profile : Gaussian or Sersic
ellipticity : 0.4
radius ratio 0.25 ~ 1.5
snr ~ 1000.0

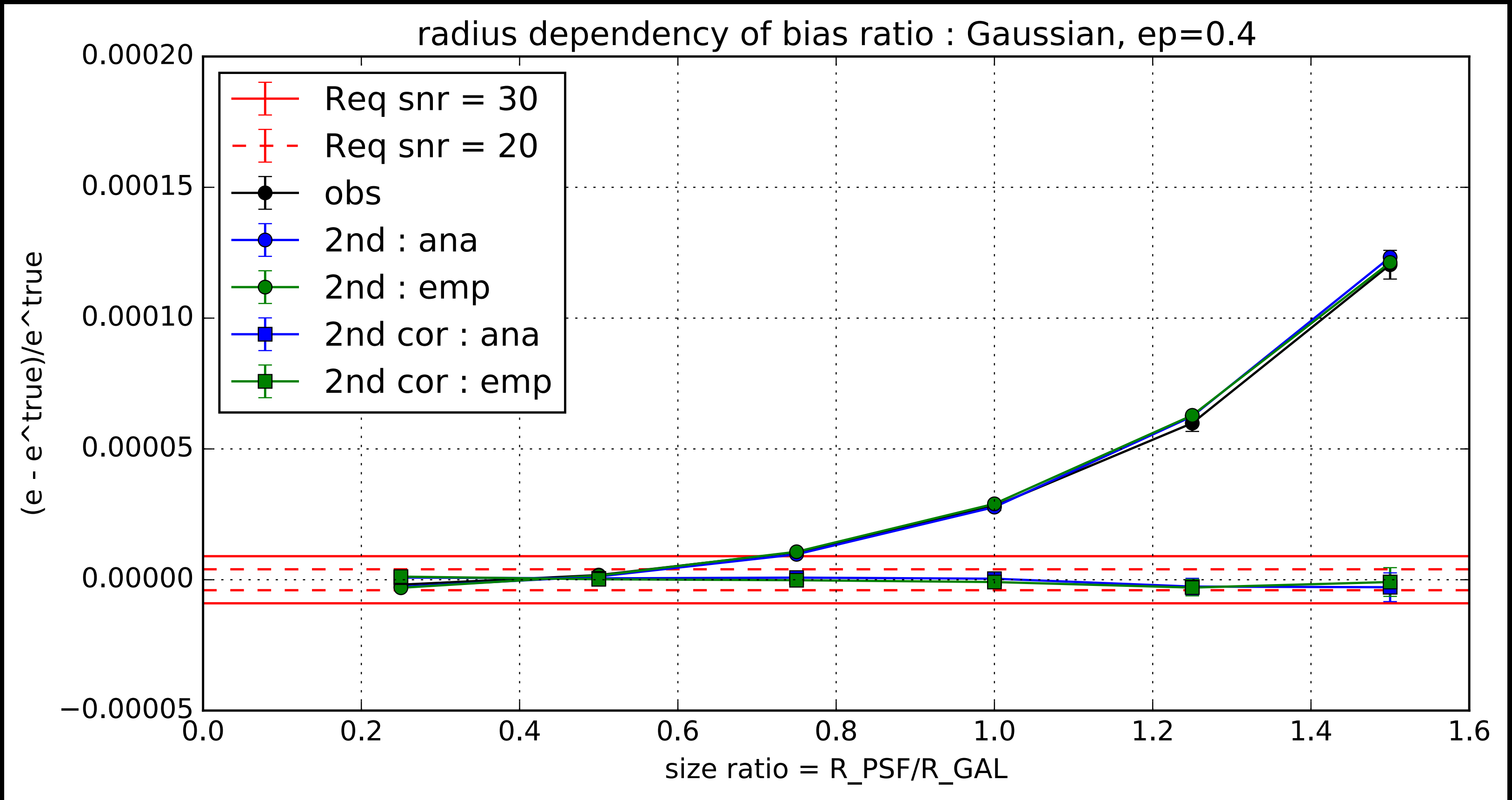
black : observed
blue : analytical (+ modification) fitting
green : empirical fitting
red : 1% error for snr = 20 or 30

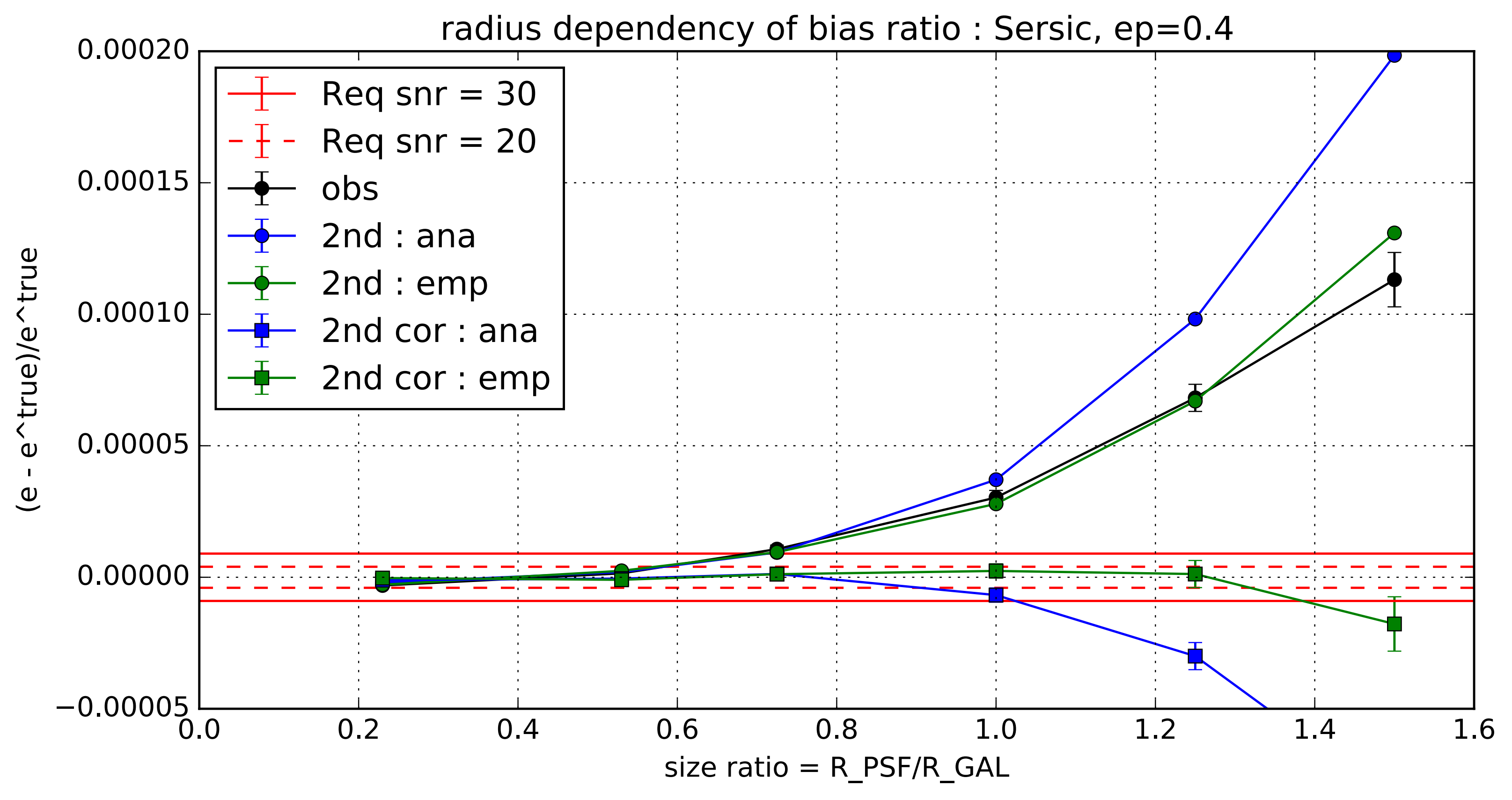
The empirical fitting is

$$\Delta\epsilon^{emp}_{(2)} = \frac{1.5R^6 + 22.0R^4 + 19R^2 - 4(1-|\epsilon|^2)}{SNR^2}$$

$$R = \frac{R_{PSF}}{R_{GAL}}\sqrt{1-|\epsilon|^2}$$

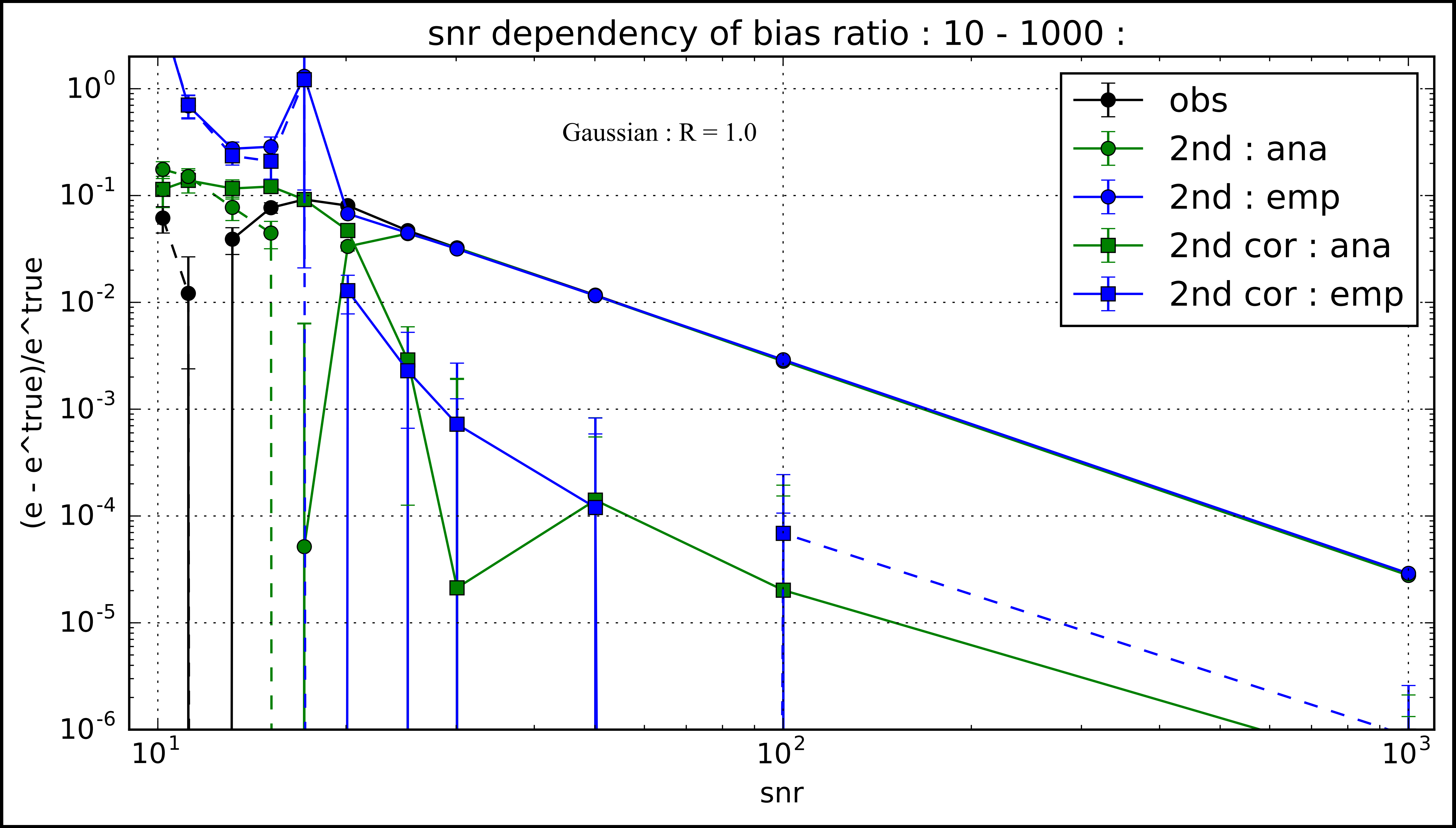
Similar results were obtained from tests with different ellipticities (0.1, 0.2, 0.3).





Simulation Test of Pixel Noise correction :

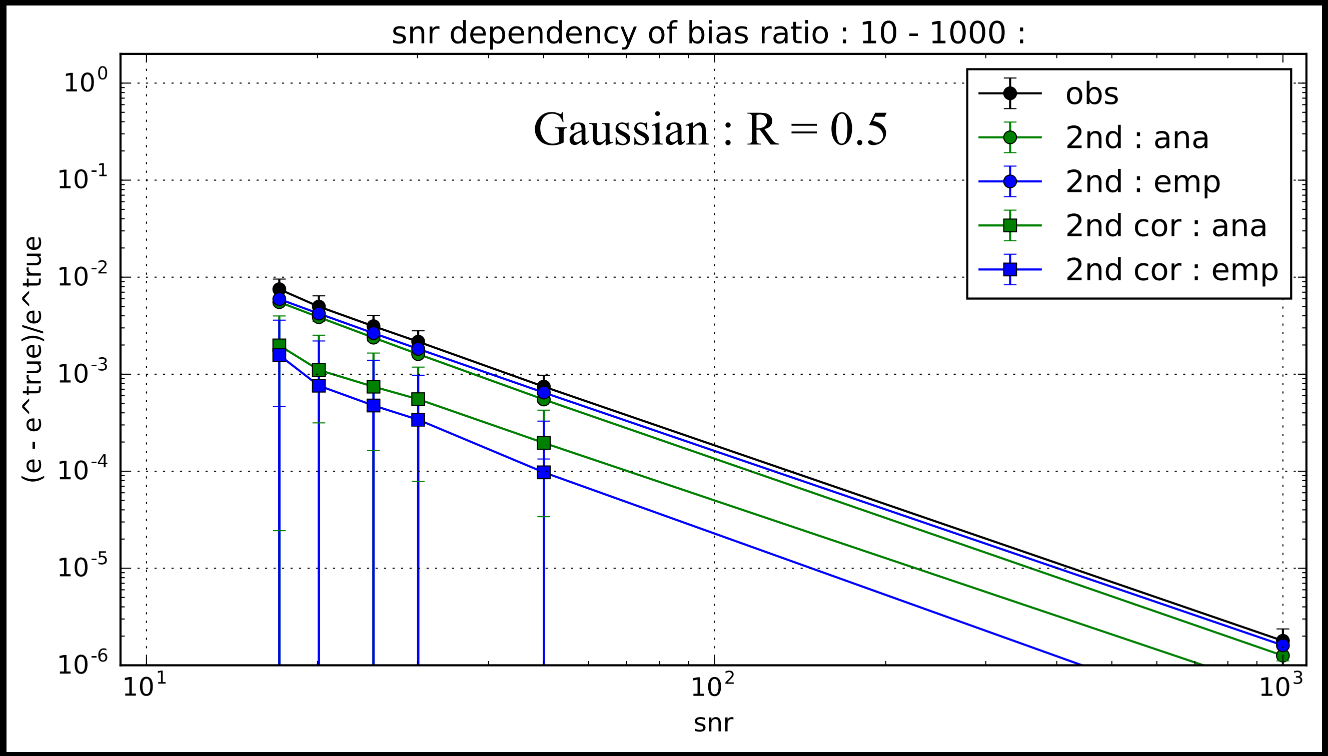
Pixel noise effect and its correction in low snr region. ellipticity : 0.4

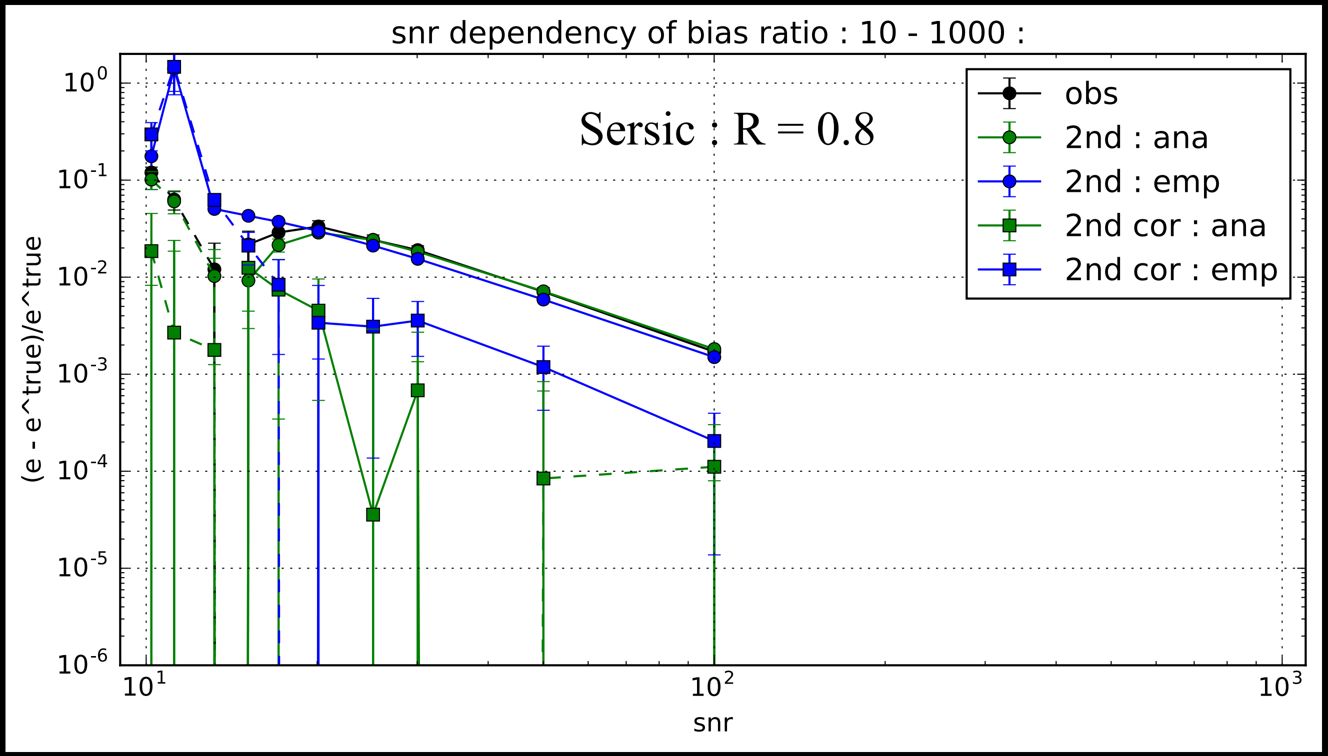


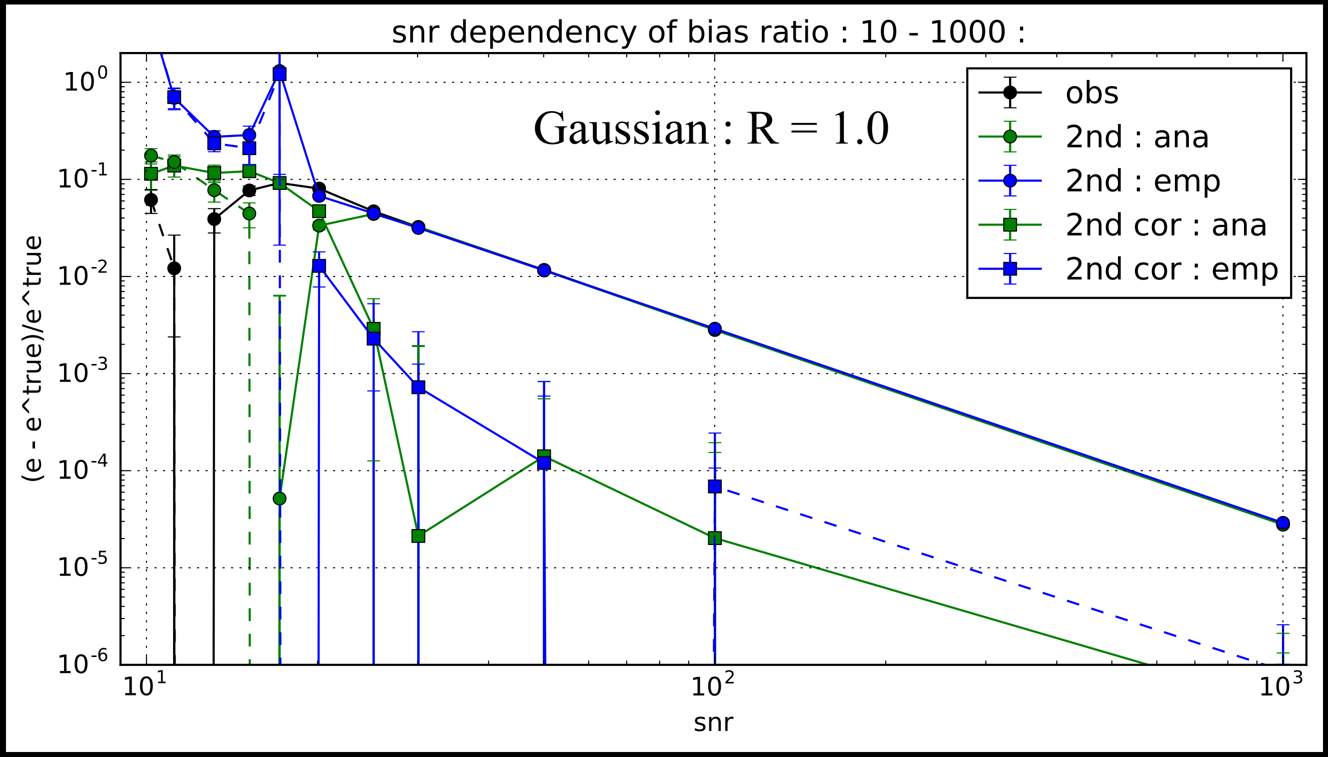
Inverse of square of snr is valid until snr ~ 25 for R=1, and the correction at lower snr is unstable.
Approximately, n(snr > 25) ~ 2.5n(snr > 50).
-> non-linear correction is needed?

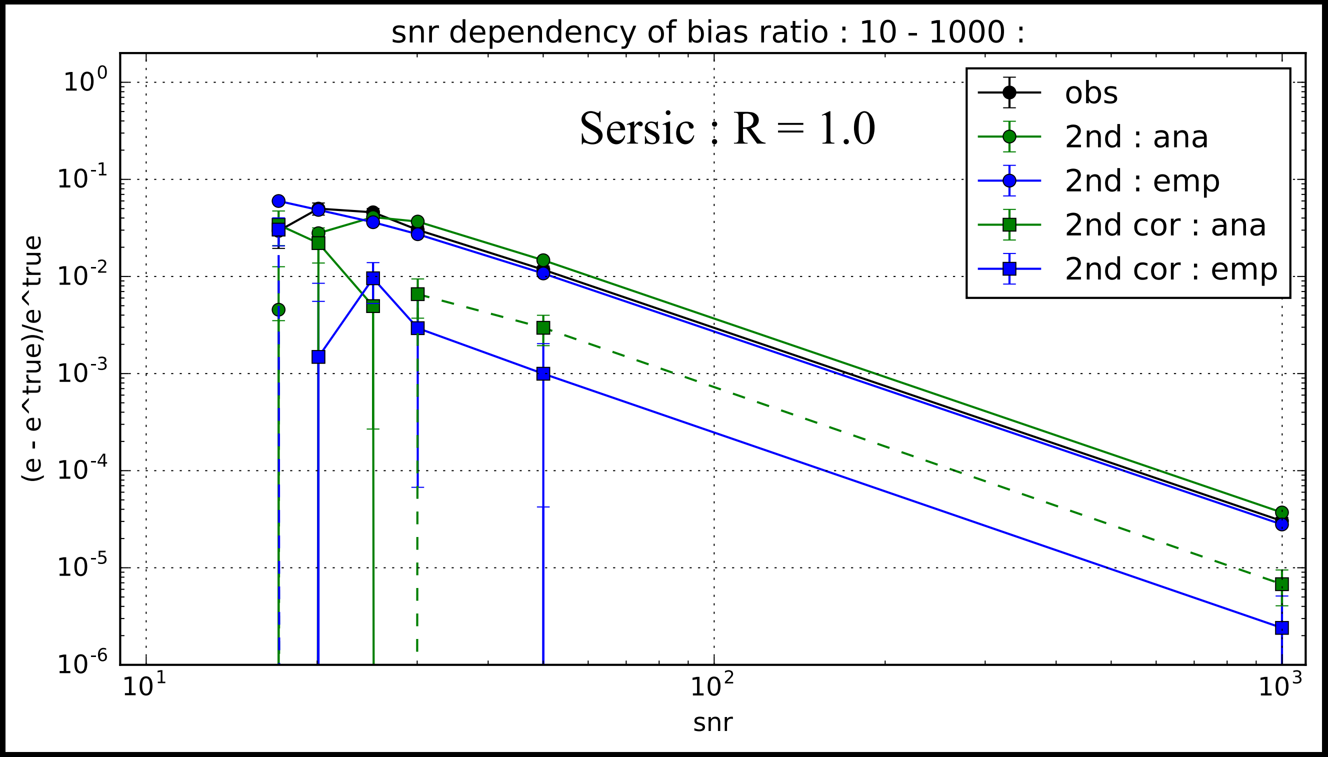
Simulation Test of Pixel Noise correction :

Same plots with different situation



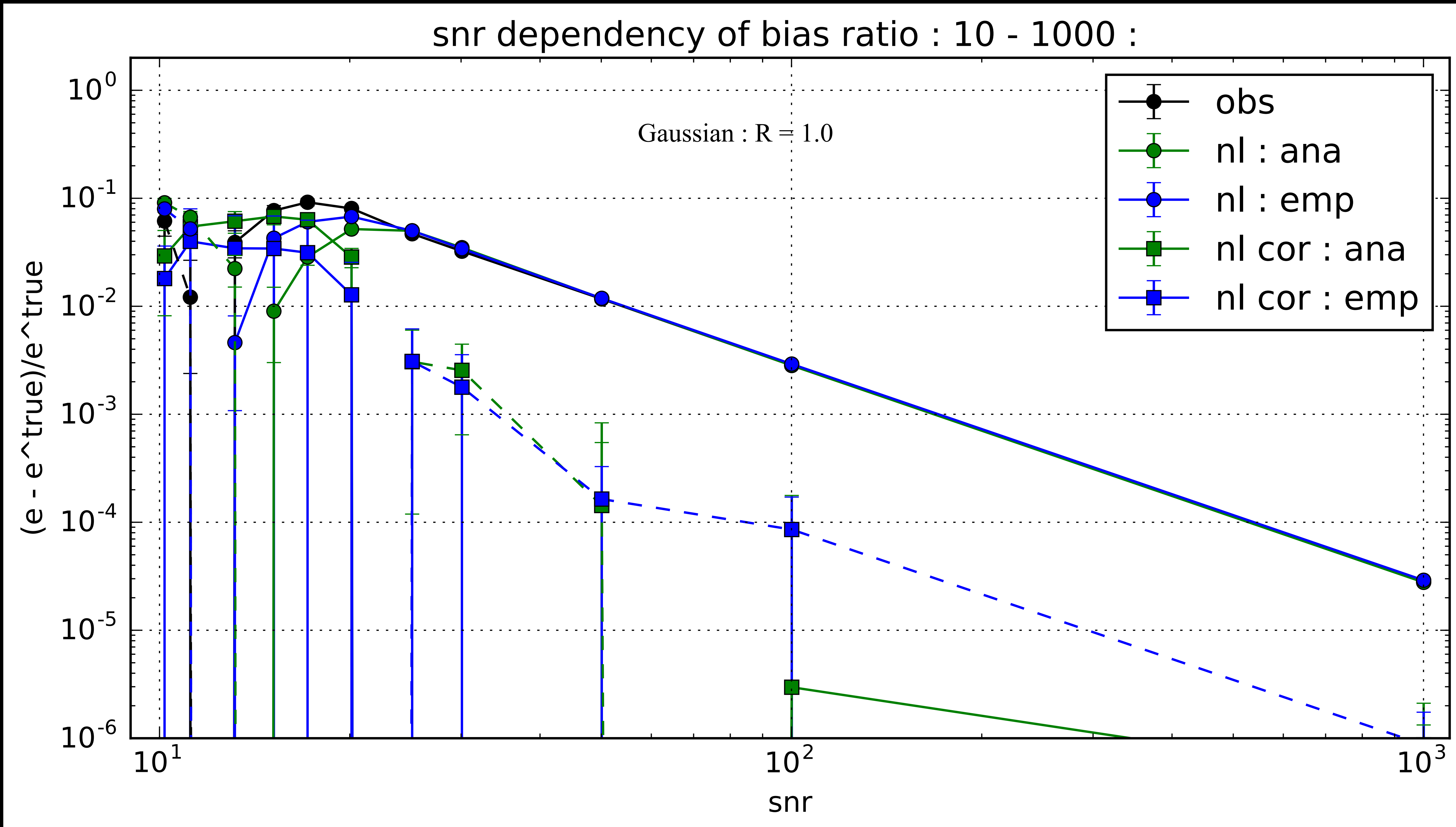






Simulation Test of Pixel Noise correction :

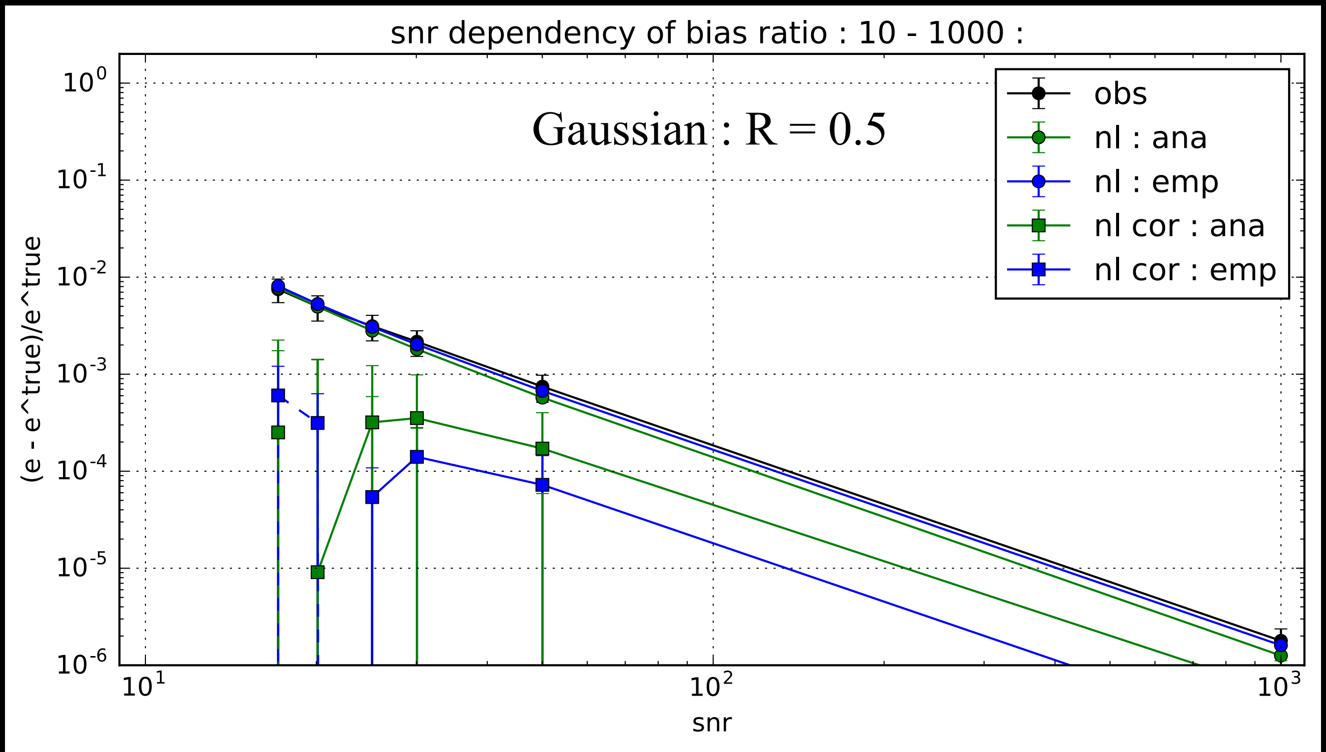
Pixel noise effect and its non-linear correction in low snr region

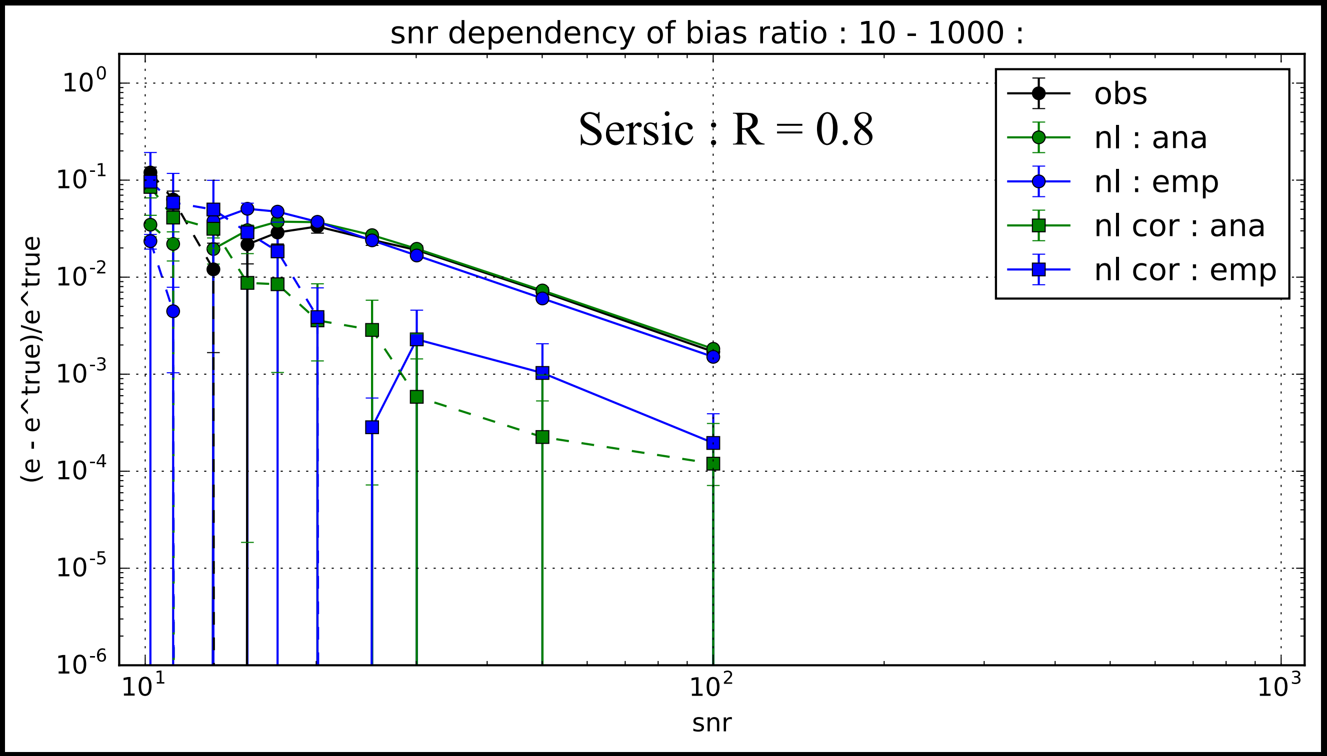


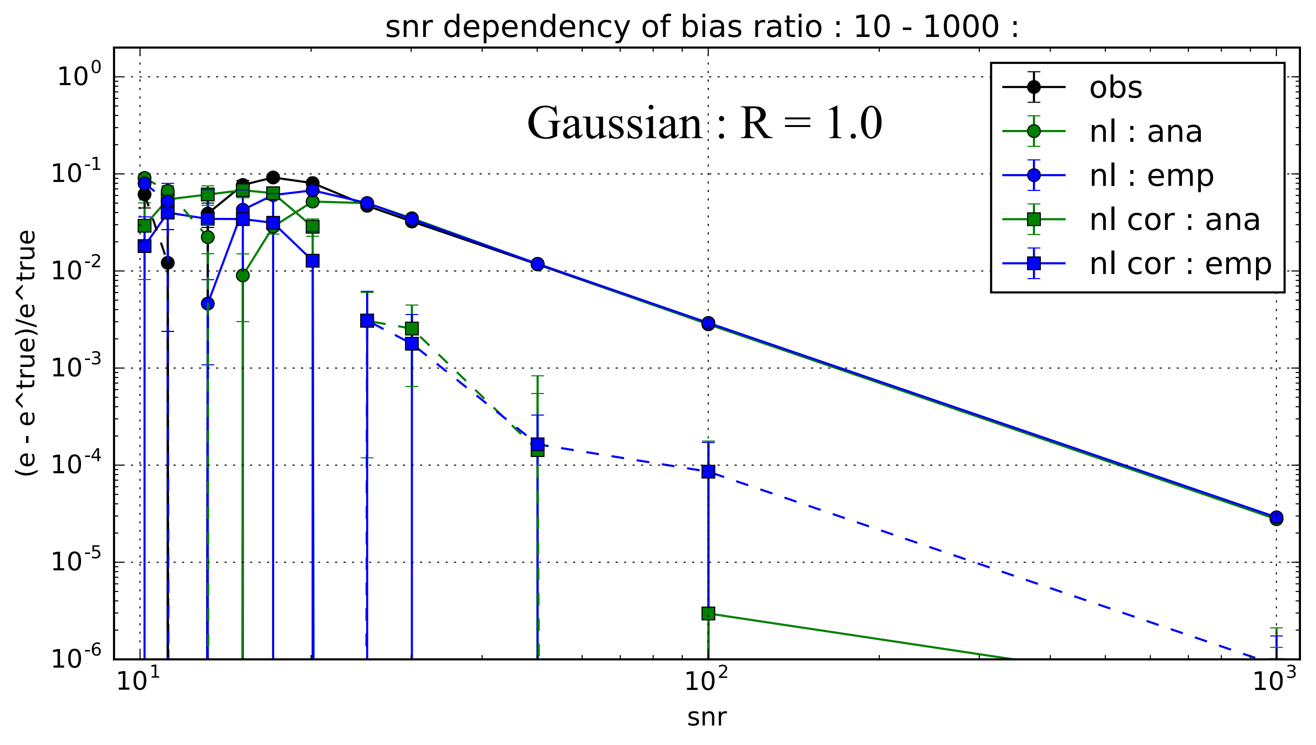
$$\Delta\epsilon_{(nl)} = \frac{1.0}{\frac{1.0}{\tanh(0.01(snr^2/R^2-50))+1} + \frac{1.0}{2.0+4.0\Delta\epsilon_{(2)}+10000R^4/snr^4}} - 1 \qquad R = \frac{R_{PSF}}{R_{GAL}}$$

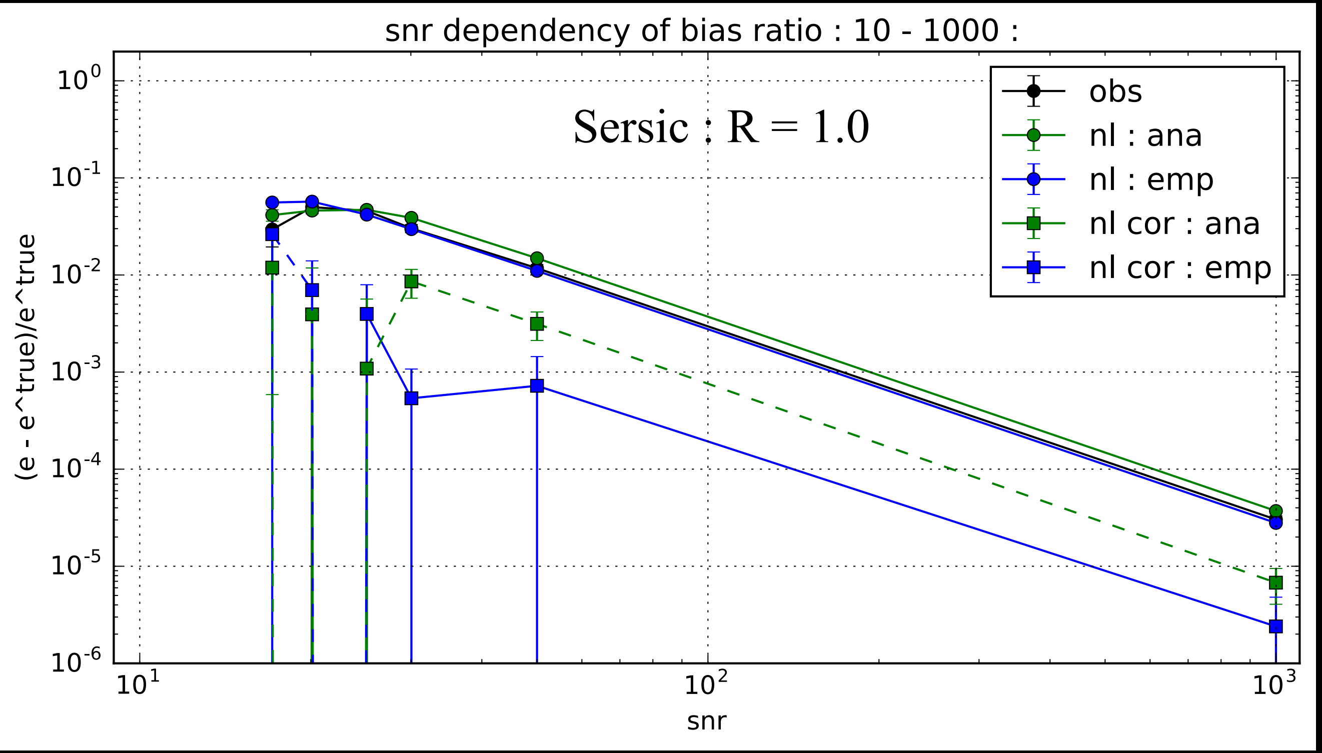
Simulation Test of Pixel Noise correction :

Same plots with different situation









The non-linear correction works well in only some cases. More studies are needed.

Summary and Future works :

Summary :

I developed a pixel noise correction method and test the correction with simple simulations.

A bias from pixel noise depends on radius ratio R and snr mainly, and independent profiles.

The bias follows inverse square law of snr in high snr region, and it reaches 1% for snr = 50, if R = 1. Then, the bias turns to negative in lower snr and goes to 0 at the limit of low snr.

The basic idea of the correction is calculating 2nd order of pixel noise effect by analytical and empirical methods, it is proportional to inverse square of snr. However, if R = 1, the inverse square low is valid until snr ~ 25 and the correction is unstable in lower snr region than 20.

Non-linear correction can correct the bias more stably, but the residuals are larger than 1% in some case.

Future works :

More investigation about the non-linear correction

Applying for HSC data (rho statistics, comparison with HSM and SDSS shapes)